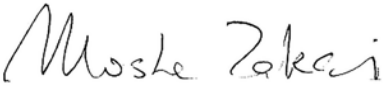

MOSHE ZAKAI

1926–2015

Elected in 1989

"For pioneering contribution to the theory of nonlinear filtering and to the theory and application of stochastic processes."

BY HAYA KASPI, EDDY MAYER-WOLF, AND OFER ZEITOUNI
SUBMITTED BY THE NAE HOME SECRETARY

MOSHE ZAKAI, who passed away November 27, 2015, in his hometown of Haifa, was an extraordinarily talented man who made a major difference in the life and career of those who collaborated with him as well as many of his students.

He was born December 22, 1926, in Sokółka, Poland, and came to Israel (then Palestine) as a child. He obtained his BSc in electrical engineering from the Technion–Israel Institute of Technology in 1951 and went to work in the scientific department of the Ministry of Defense as a radar engineer until 1956. With a government fellowship, he obtained a PhD in electrical engineering in 1958 from the University of Illinois at Urbana-Champaign and then returned to the Ministry of Defense scientific department as head of the Communication Research Group. In 1965 he joined the Faculty of Electrical Engineering at the Technion, where he remained until his retirement in 1998 as a Distinguished Professor.

Moshe Zakai strongly felt that it was essential to use modern advanced mathematical tools in the study of communication and radar theory. Soon after his PhD he took a keen interest in

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Kiyosi Itô's stochastic integration theory and in stochastic differential equations as the proper model for dynamical systems driven by white noise. Together with Eugene Wong, he realized that there was a serious obstacle in applying Itô's theory: white noise is not physical, and Itô's solution was not continuous in the input, in the sense that driving a stochastic differential equation with an approximation of white noise does not yield a solution that is close to Itô's solution.

In a groundbreaking 1965 paper,[1] Wong and Zakai showed how to resolve this problem: an extra term (now called the Wong-Zakai correction) has to be added to the "physical" equation, and with it continuity is restored. This observation opened the door to rigorous applications of the Itô calculus in communication and control on the one hand, and new developments in the theory of stochastic processes on the other. To some extent, one could interpret Martin Hairer's more recent theory of regularity structures (for which he received the Fields Medal in 2014) as a suitable way to introduce Wong-Zakai corrections in the more challenging setup of nonlinear stochastic partial differential equations.

Another topic to which Zakai made a seminal contribution is the theory of nonlinear filtering. Filtering deals with extracting a signal from a noisy observation of it by computing the conditional distribution of the signal given the observations. In the setup of Gaussian processes, the problem was solved in the 1940s by Norbert Wiener and Andrey Kolmogorov. (It is worth noting that Wiener was motivated by control applications stemming from the World War II effort.) Later, Rudolf Kálmán devised a recursive filter that computed the optimal (linear) filter; his filter was a crucial element in the development of modern control, radar, and communication systems. However, it did not always approximate well the optimal filter for non-Gaussian models, which is generally nonlinear.

The mid-1960s saw a flurry of activity in addressing this challenge, and various representations of the optimal filter

[1] On the Convergence of Ordinary Integrals to Stochastic Integrals. *Annals of Mathematical Statistics* 36(5):1560–1564.

were derived. However, none could be computed effectively, as the problem required solving an infinite system of coupled stochastic differential equations. Zakai's major insight in his fundamental 1969 paper[2] was to realize that by focusing on an unnormalized version of the conditional density, one could obtain a single bilinear stochastic partial differential equation (the Zakai equation), from which the filter could be easily computed (and which reduces to the Kalman filter in the Gaussian case). Zakai's equation has been the basis for all progress in filtering theory; in particular, modern approaches to compute the filter using genetic algorithms ("particle filters") effectively compute the solution to Zakai's equation.

After a foray with Wong into the study of multiparameter stochastic processes, the last two decades of Zakai's professional life saw the completion of his transition to a full-time probabilist. He turned his attention to the Malliavin calculus, introduced by Paul Malliavin in 1979 to study the smoothness of Gaussian functionals—in particular of solutions to Itô equations—with respect to perturbations of the driving white noise, with the aim of providing a probabilistic proof of Lars Hörmander's criterion for the regularity of solutions of parabolic partial differential equations. Zakai was one of a handful of probabilists who started working on Malliavin's calculus shortly after its introduction. Very early on, he introduced a different, more geometric, approach summarized in his influential 1985 paper.[3]

His old concern with the continuity of functionals in the underlying white noise resurfaced in 1990 in one of his many joint papers with David Nualart, now in a more abstract setting, identifying the multiple Wiener integrals, which are continuous in Brownian motion. He then embarked with A. Süleyman Üstünel and others on a program to apply these ideas to the study of anticipative changes of measures on Wiener space.

[2] On the Optimal Filtering of Diffusion Processes. *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete* 11(3):230.

[3] Malliavin Derivatives and Derivatives of Functionals of the Wiener Process with Respect to a Scale Parameter. *Annals of Probability* 13(2): 609–615.

Their joint book from 2000[4] summarizes the theory and is the standard reference for the study of transformations on Wiener space.

Zakai's work was recognized by many awards, including the Control Systems Award from the Institute of Electrical and Electronics Engineers (IEEE) and Israel's Rothschild Prize. He was a fellow of the IEEE and Institute of Mathematical Statistics, foreign associate of the US National Academy of Engineering, and member of the Israel Academy of Sciences and Humanities.

He was a strong proponent of employing sophisticated mathematical tools in engineering, and showed by example that the interaction between mathematics and engineering is highly beneficial to both disciplines. His voice and mentorship will be sorely missed by his many colleagues, students, and friends.

He is survived by his wife Shulamit (Mita); their children Tamar, Michal, and Noam; and grandchildren and great-grandchildren.

[4] *Transformation of Measure on Wiener Space*. Berlin/Heidelberg: Springer-Verlag.